Robust Optimization in Electric Power Systems

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1 Introduction

Electric power systems, acclaimed as the "supreme engineering achievement of the 20th century", are one of the most complex human-made systems [9]. As an example, the U.S. power grid has approximately 170,000 miles of high-voltage (voltage at or above 200kV) transmission lines and almost 5,000 generating units with capacity of at least 50MW [1]. In an engineering system of such a scale and complexity, uncertainty abounds. More specifically, uncertainties in generation, consumption, and unexpected failures of electric equipments have to be carefully considered in the operation of power systems. It is an amazing fact that the reliability and security of power supply is held to such a high standard that flipping on a switch and expecting the light bulbs to shine at any time has almost become a part of the subconsciousness of the modern society. This is a great achievement, made possible by the sound engineering design and the tremendous efforts dedicated by the electricity industry.

However, it becomes increasingly challenging to uphold such a high standard of reliability, as the power systems around the world experience fast and fundamental changes. In particular, the output of generators, which traditionally has been well predictable, is becoming more and more difficult to predict. The key driving force behind this change is the large-scale integration of wind and solar power generation into the power grids, both of which are highly intermittent, correlated in time and space, and stochastic in nature [2]. At the same time, the demand side is also becoming more and more intelligent and responsive, with the smart grid technologies enabling electricity consumers to change their consumption in real time. All of these changes have significant implications on power systems at multiple levels.

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Figure 1 shows a picture of the major decision-making problems involved in power systems operation. The problems are categorized from a temporal perspective into real-time operations (minute-to-minute dispatch), daily scheduling (day-ahead commitment), midterm maintenance planning (seasonal or annual schedules), and long-term investment planning for generation and transmission systems [24]. In an electricity market environment, the independent system operator (ISO) is responsible for scheduling dispatch and daily commitment, as well as coordinating maintenance schedules between generation and transmission owners [25]. The ISO would also conduct long-term planning study and make technical recommendations for generation and transmission expansion. At the same time, all of these decision-making problems involve numerous stakeholders and affect millions of consumers. Mathematical optimization is heavily relied upon for reaching consistent, efficient, and optimal outcomes. It is not an exaggeration to say that modern power systems are built upon rigorous and sound optimization models and efficient solution algorithms.

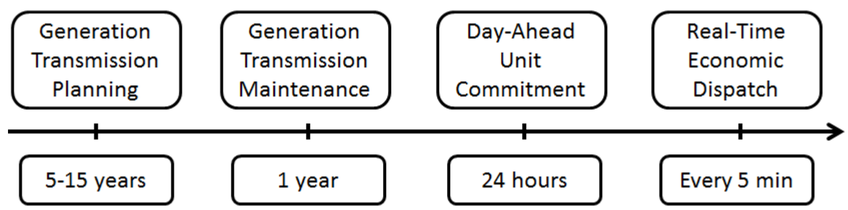

Figure 1: Major decision-making problems in power systems operation. The time arrow points to the start of real-time operation. The typical decision horizon of each problem is given under each problem [28].

Traditionally, the above decisions-making problems are solved by deterministic optimization models [32]. That is, the optimization models consider a problem of fixed and known parameters. For example, in the short-term economic dispatch and unit commitment, the generation capabilities, the exact demand at each of the future periods, and the conditions of the transmission lines and generators are assumed to be known. The resulting solutions are therefore feasible and optimal for the planned or forecast scenario. Such a deterministic approach has been successful at maintaining power systems reliability and security under the traditional conditions. However, as the uncertainties in the power systems multiply due to the growing penetration of renewable resources, the traditional approaches are becoming inadequate for the multi-leveled decision-making problems. The industry is actively seeking new approaches to deal with the growing uncertainties (see e.g. [6]).

Recently, robust optimization has been developed into a rich and practical methodology for decision-making under uncertainty [5]. In the past few years, a flurry of research activities has introduced robust optimization into the field of power systems [36, 15, 6, 17]. In a short amount of time, interesting models are proposed and promising results are shown to demonstrate the power of robust optimization in almost every category of the decision problems depicted in Figure 1. The industry has taken the lead in supporting and pursuing collaborations with academic researchers. The present chapter aims to give a concise overview of the recent advances in the field. The review is bound to be incomplete due to the fast growth of this area.

This chapter first focuses on the unit commitment problem in the day-ahead operation, which is a building block for many other decision-making problems in power systems; then we discuss real-time and long-term planning. In particular, Section 2 introduces the two-stage robust optimization based unit commitment model, solution methods, and some computational results. Section 3 presents several extensions to this fundamental model. Section 4 discusses some recent progress on robust optimization models for real-time operation and long-term planning. Section 5 closes the chapter with some discussions on future directions.

2 Two-stage adaptive robust optimization for security-constrained unit commitment problem

In this section, we first present the deterministic security-constrained unit commitment (SCUC) model, which is used by most of the system operators in the day-ahead scheduling. Then, a fundamental SCUC model based on two-stage adaptive robust optimization is discussed.

2.1 Deterministic security-constrained unit commitment

Unit commitment (UC) is a process of determining the on and off status of generation units (primarily the thermal units, i.e., coal, nuclear, geothermal, and natural gas power plants) and their production levels for next day operation. The scheduling horizon is usually 24 hours or 36 hours with an hourly interval. The on and off decision is called the unit commitment decision. The production level is called the dispatch decision. The production of the generation units is scheduled to meet the forecast demand in the power network, satisfying various physical and operational constraints. To ensure a certain level of security, the system operator usually requires the UC and dispatch solutions to be feasible for any one failure of a generation unit and/or a transmission line. Such constraints are called $N-1$ security constraints. A typical deterministic SCUC model with transmission security constraints is presented below [28].

$$
\begin{aligned}
\min_{x,u,v,p} \quad & \sum_{t\in\mathscr{T}}\sum_{i\in\mathscr{G}}\Big(f_i^t(x_i^t,u_i^t,v_i^t)+c_i^t(p_i^t)\Big) && \text{(1a)}\\
\text{s.t.} \quad & x_i^{t-1}-x_i^t+u_i^t\geq 0, \quad \forall i\in\mathscr{G}, t\in\mathscr{T}, && \text{(1b)}\\
& x_i^t-x_i^{t-1}+v_i^t\geq 0, \quad \forall i\in\mathscr{G}, t\in\mathscr{T}, && \text{(1c)}\\
& x_i^t-x_i^{t-1}\leq x_i^\tau, \quad \forall \tau\in[t+1,\min\{t+\mathrm{MinUp}_i-1,T\}], && \\
& \qquad\qquad\qquad\qquad\qquad\qquad\qquad t\in[2,T], i\in\mathscr{G}, && \text{(1d)}\\
& x_i^{t-1}-x_i^t\leq 1-x_i^\tau, \quad \forall \tau\in[t+1,\min\{t+\mathrm{MinDw}_i-1,T\}], && \\
& \qquad\qquad\qquad\qquad\qquad\qquad\qquad t\in[2,T], i\in\mathscr{G} && \text{(1e)}\\
& \sum_{i\in\mathscr{G}}p_i^t=\sum_{j\in\mathscr{D}}\bar{d}_j^t, \qquad \forall t\in\mathscr{T}, && \text{(1f)}\\
& p_i^t-p_i^{t-1}\leq RU_i x_i^{t-1}+SU_i u_i^t, \quad \forall i\in\mathscr{G}, t\in\mathscr{T}, && \text{(1g)}\\
& p_i^{t-1}-p_i^t\leq RD_i x_i^t+SD_i v_i^t, \quad \forall i\in\mathscr{G}, t\in\mathscr{T}, && \text{(1h)}\\
& -f_{l,k}^{\max}\leq \boldsymbol{a}_{l,k}^{\mathsf{T}}(\boldsymbol{p}^t-\boldsymbol{d}^t)\leq f_{l,k}^{\max}, \quad \forall t\in\mathscr{T}, l\in\mathscr{C}_k, k\in\mathscr{L}, && \text{(1i)}\\
& p_i^{\min}x_i^t\leq p_i^t\leq p_i^{\max}x_i^t, \quad \forall i\in\mathscr{G}, t\in\mathscr{T}, && \text{(1j)}\\
& x_i^t,u_i^t,v_i^t\in\{0,1\}, \quad \forall i\in\mathscr{G}, t\in\mathscr{T}. && \text{(1k)}
\end{aligned}
$$

The unit commitment decisions include binary variables x_i^t, u_i^t, v_i^t, where $x_i^t=1$ if generator i is on at time t, and $x_i^t=0$ otherwise; $u_i^t=1$ if generator i is *turned* on from the off state at time t; $v_i^t=1$ if generator i is turned off at time t. The dispatch decision is p_i^t of generator $i\in\mathscr{G}$ at time $t\in\mathscr{T}$, where $\mathscr{G}$ is the set of generators, and $\mathscr{T}$ is the set of time periods in the decision horizon.

The fixed cost $f_i^t(x_i^t,u_i^t,v_i^t)$ of each generator includes start-up and shut-down costs and other fixed costs. The variable cost $c_i^t(p_i^t)$ is usually approximated by a convex piecewise linear function of the active power output p_i^t. The forecast demand $\bar{d}_j^t$ is the load at bus j, time t. Constraints (1b) and (1c) represent logic relations between on and off status and the turn-on and turn-off actions. Constraints (1d) and (1e) restrict the minimum up and down times for each generator. Constraint (1f) enforces system wide energy balance in each time period. Constraints (1g)-(1h) limit the rate of production changes over a single period, where RU_i and RD_i are limits for ramp-up and ramp-down rates when the generator is already running, and SU_i and SD_i are ramping limits when generator i is just starting up and shutting down. Constraint (1i) expresses the power flow in the transmission lines as a linear function of power production and load in the entire system, where the coefficients of the linear function, $a_{l,k}$, are called the shift factors of line l, and the index k represents the k-th contingency, i.e., when the line k is tripped offline, $\mathscr{C}_k$ is the set of remaining lines. Constraint (1j) represents the physical limits on the production levels of each generator.

The above SCUC model (1) is the basic formulation for a UC model. An important aspect that is not presented in (1) is the system reserve and associated constraints [3]. The reliability of power supply is so critical for the function of modern society that system operators pay extremely careful attention to ensure the system has enough generation capacity. The industry practice is to use the so-called reserves, generation resources that are online or can be quickly brought online to respond to any demand surge or generation outages. The system-wide reserve requirement is usually pre-determined by certain rules, such as a percentage of the forecasted peak load plus the largest online generator's capacity. Then, the reserve levels of individual generators are co-optimized with the UC and dispatch decisions [8, 7, 11]. In short, reserve is an engineering way to cope with uncertainties. In the following, we will present robust optimization based SCUC models, which in a sense rigorously quantify uncertainties and replace or reduce the ad-hoc reserve requirement.

2.2 Adaptive robust UC model with net load uncertainty

As discussed in Section 1, the main sources of uncertainties in the day-ahead unit commitment include the demand uncertainty, renewable generation uncertainty, and unexpected failures of transmission lines and generators. In this subsection, we first present a fundamental robust optimization model that considers the uncertainty in the net load, where the net load is demand minus renewable generation. Then, we present several generalizations. Consider the following robust UC model [36, 15, 6, 17]:

$$\begin{aligned} \min_{x} \quad & \left\{ c(x) + \max_{d \in \mathscr{D}} \min_{p \in \Omega(x,d)} b(p) \right\} \\ \text{s.t.} \quad & x \in \mathscr{F}, \end{aligned} \tag{2}$$

where x is the vector of commitment related decisions, p is the vector of dispatch variables, and d is the vector of net load in the network; $\mathscr{D}$ is a set that describes the region of possible net load; $\Omega(x,d)$ is the feasible region of the dispatch problem, defined by (1f)-(1j). More compactly, we can write it as $\Omega(x,d) = \{p : Hp + Ed \le h, Ax + Bp \le g\}$, where the first linear inequality represents the constraints (1f) and (1i), involving the dispatch p and demand d, and the second linear inequality represents the constraints (1g), (1h), and (1j). The set $\mathscr{F}$ is the feasible region of the commitment decisions x, which is defined by (1b)-(1e).

This formulation (2) is a two-stage fully adaptive robust optimization model. The commitment decision $\boldsymbol{x}$ is the first-stage decision, made before the realization of the uncertain net load $\boldsymbol{d}$, whereas the dispatch decision $\boldsymbol{p}$ is the second-stage decision taken to respond to each specific realization of $\boldsymbol{d}$. That is, the dispatch solution $\boldsymbol{p}(\boldsymbol{b})$ as a function of $\boldsymbol{b}$ fully adapts to any $\boldsymbol{b}$. The solution of (2) is a unit commitment decision $\boldsymbol{x}$ that is feasible, therefore robust, for any possible realization of the uncertain net load $\boldsymbol{d}$.

The *uncertainty set* $\mathscr{D}$ describes the ranges of possible net load $\boldsymbol{d}$. We can also impose some constraints on the total variations of the uncertainty in $\mathscr{D}$ and use it to control the conservativeness of the robust model. The following so-called budgeted uncertainty set is widely used in the literature. For each time t:

$$\mathscr{D}^t(\bar{\boldsymbol{d}}^t, \hat{\boldsymbol{d}}^t, \Delta^t) := \left\{ \boldsymbol{d}^t : \sum_{i \in \mathscr{N}_d} \frac{|d_i^t - \bar{d}_i^t|}{\hat{d}_i^t} \leq \Delta^t, d_i^t \in \left[\bar{d}_i^t - \hat{d}_i^t, \bar{d}_i^t + \hat{d}_i^t\right], \forall i \right\}, \quad (3)$$

where $\mathscr{N}_d$ is the set of nodes that have uncertain net load, $\boldsymbol{d}^t = (d_i^t, i \in \mathscr{N}_d)$ is the vector of uncertain net load at time t, $\bar{d}_i^t$ is the nominal value of the net load of node i at time t, $\hat{d}_i^t$ is the deviation from the nominal net load, and the interval $\left[\bar{d}_i^t - \hat{d}_i^t, \bar{d}_i^t + \hat{d}_i^t\right]$ is the range of the uncertain d_i^t. The inequality in (3) controls the deviation of total net load from the nominal value. The parameter Δ^t is the "budget of uncertainty". With $\Delta^t = 0$, the uncertainty set $\mathscr{D}^t = \{\bar{\boldsymbol{d}}^t\}$ is a singleton, corresponding to the nominal deterministic case. As Δ^t increases, the size of the uncertainty set $\mathscr{D}^t$ enlarges. This means that larger total deviation from the expected net load is considered, so that the resulting robust UC solutions are more conservative and the system is protected against a higher degree of uncertainty. With $\Delta^t = N_d$, where N_d is the total number of uncertain net load, $\mathscr{D}^t$ equals to the entire hypercube defined by the intervals for each d_i^t.

The uncertainty set in (3) is defined independently for each time period. A budget constraint over all the time periods can also be added to limit the variation of net loads over the entire planning horizon. The basic structure of the budgeted uncertainty sets remain the same. In the later part, we will introduce another type of uncertainty sets which model the dynamics of uncertainty parameters between time periods and locations.

2.3 Solution method to solve adaptive robust model

The two-stage robust optimization model (2) is a general and fundamental model. The complexity of solving such a model comes from two sources. One is from the discrete nature of the first-stage decision x. The other is from the max-min structure in the second stage. The latter one is more fundamental in the sense that even if the first-stage decision is continuous with convex region $\mathscr{F}$, the second-stage problem can still be computationally challenging. To see this, we can reformulate the second-stage max-min problem by the strong duality of linear optimization, assuming linear cost functions:

$$\begin{aligned}
&\max_{d\in\mathscr{D}} \min_{p} \left\{ \boldsymbol{b}^\top \boldsymbol{p} : \boldsymbol{H}\boldsymbol{p} + \boldsymbol{E}\boldsymbol{d} \le \boldsymbol{h}, \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{p} \le \boldsymbol{g} \right\} \\
=&\max_{d,\mu,\eta} \ \boldsymbol{d}^\top \boldsymbol{E}^\top \boldsymbol{\mu} - \boldsymbol{h}^\top \boldsymbol{\mu} - (\boldsymbol{g} - \boldsymbol{A}\boldsymbol{x})^\top \boldsymbol{\eta} && \text{(4a)} \\
&\quad \text{s.t. } \boldsymbol{H}^\top \boldsymbol{\mu} + \boldsymbol{B}^\top \boldsymbol{\eta} = \boldsymbol{b}, \ \boldsymbol{\mu}, \boldsymbol{\eta} \ge 0 && \text{(4b)} \\
&\qquad\quad \boldsymbol{d} \in \mathscr{D} && \text{(4c)}
\end{aligned}$$

Notice that the objective function (4a) has a bilinear term $\boldsymbol{d}^\top \boldsymbol{E}^\top \boldsymbol{\mu}$, and the feasible region is composed of two separate polyhedrons for $(\boldsymbol{\mu}, \boldsymbol{\eta})$ and $\boldsymbol{d}$, respectively. By the strong duality of linear optimization, the above procedure can also be reversed so that a bilinear optimization problem with two separate polyhedral feasible regions can be reformulated as a max-min problem as in the second-stage of (2). In general, this type of bilinear optimization problem is NP-hard to solve [4], which indicates the second-stage problem of (2) is computationally challenging, independent of the first-stage problem.

However, it is interesting to note that the optimal objective value of the max-min problem (4) denoted as $R(\boldsymbol{x})$ is a convex function of the first-stage decision $\boldsymbol{x}$. This suggests that, modulo the complexity of the second-stage problem, the overall two-stage problem (2) may be reasonably solvable by a Benders decomposition type algorithm, which is developed in [6, 15, 17].

Another key property of the second-stage problem is that the worst-case net load is always an extreme point of the uncertainty set $\mathscr{D}$. This follows from the well-known property of bilinear optimization problems with separate polyhedron sets [18]. With this observation, the two-stage robust model (2) can be re-written as

$$\begin{aligned}
&\min_{x\in\mathscr{F},z,p} \ c(\boldsymbol{x}) + z && \text{(5)} \\
&\qquad \text{s.t. } z \ge \boldsymbol{b}^\top \boldsymbol{p}_k \quad \forall k = 1, \ldots, m \\
&\qquad\qquad \boldsymbol{p}_k \in \Omega(\boldsymbol{x}, \boldsymbol{d}_k) \quad \forall k = 1, \ldots, m
\end{aligned}$$

where $(\boldsymbol{d}_1, \ldots, \boldsymbol{d}_m)$ is the set of extreme points of the polyhedron $\mathscr{D}$. The number of extreme points may be exponential in the dimension of $\mathscr{D}$, which is the case of the budget uncertainty (3). This presents an ideal situation to apply constraint generation on (5). Since new variables p_k are also generated with the new constraints, such an algorithm is formally proposed in [34] with the name, column-and-constraint generation (see also [30]). A similar procedure is also proposed in [6] as a heuristic to speed up the Benders decomposition, where the worst-case extreme points of $\mathscr{D}$ together with the associated dispatch constraints are added to the first-stage problem in each iteration of the Benders algorithm. For details, please refer to [34, 17, 6]. The recent work in [19] proposes efficient algorithms to deal with transmission constraints in (2), which dynamically include critical transmission lines.

2.4 Computational study

The two-stage robust SCUC model (2) with the budgeted uncertainty sets (3) has been applied to the day-ahead scheduling of the ISO New England's power system [6]. Table 1 shows the comparison of average dispatch and total costs between the two-stage robust SCUC and the current practice of deterministic UC with reserve adjustment [6].

	Two-Stage Robust		Reserve Adjustment	
budget $\Delta^t/\sqrt{N_d}$	dispatch cost (M$)	total cost (M$)	dispatch cost (M$)	total cost (M$)
0.5	16.9195	18.6050	18.1855	19.6837
1.0	16.9650	18.6688	17.4907	18.9942
1.5	16.9815	18.7365	17.3027	18.8006
2.0	17.0297	18.7937	17.7403	19.2415
2.5	17.0586	18.8366	17.6567	19.1618
3.0	17.0745	18.8526	18.0804	19.5889

Table 1: The average dispatch and total costs of the two-stage robust UC and the deterministic UC with reserve for normally distributed net load $\Delta^t/\sqrt{N_d} = 0.5, 1, \ldots, 3$ and $\hat{d}_j^t = 0.1\bar{d}_j^t$.

From this table, we can see that both dispatch and total costs are reduced by the two-stage robust UC model. Also it is worth noticing that the lowest average cost is achieved at an uncertainty budget $\Delta^t = 0.5\sqrt{N_d}$, which results in an uncertainty set that is much smaller than the box uncertainty set of the net load intervals in (3). The computational study also shows that the robust UC model can significantly reduce the variability of the production cost by more than an order of magnitude [6].

3 Extensions to two-stage robust UC models

In this section, we present several recent extensions to the two-stage robust SCUC model (2).

3.1 Security-constrained UC with corrective actions

The UC model (1) finds a commitment and dispatch solution that is feasible for any one transmission line contingency. This is called the $N-1$ security constrained UC with preventive actions. A more stringent requirement is to prepare for any loss of k transmission lines, i.e., the $N-k$ security constrained UC. Furthermore, a more flexible way to respond to contingencies is to allow the commitment and dispatch solution to change, the so-called SCUC with corrective actions. It is easy to see that $N-k$ security-constrained UC with corrective actions is a robust optimization model with decisions adaptive to contingency uncertainties. The seminal work [27] presents such a robust UC model for SCUC with generation contingency, i.e., any k online generators may experience unexpected failures in each period and remaining generators can be re-committed and re-dispatched. A subsequent work [30] proposes an elegant robust UC model that considers both generation and transmission line contingencies in a power network. A similar two-stage robust UC model is proposed in [26], which considers both generation and transmission contingencies and reserve scheduling. Notice that net load uncertainty is not explicitly considered in these models.

3.2 Taming the conservativeness

The adaptive robust UC model (2) minimizes the sume of the commitment cost and the worst-case second-stage dispatch cost. As shown in [6], the balance between the conservativeness and the robustness of the resulting UC solution can be controlled by the budget constraints in (3). A nice discussion on the modeling choices of uncertainty sets and their implications on conservativeness of the UC solutions is provided in [12]. To further tame the conservatiness, alternative objective functions are considered. For example, a regret optimization based approach is proposed in [16], and a hybrid model minimizing the expected cost and the worst-case cost is proposed in [35].

The two-stage minimax regret robust UC model can be presented abstractly as $\min_{x\in\mathscr{F}} \{\mathrm{Reg}(\boldsymbol{x}) : \Omega(\boldsymbol{x},\boldsymbol{d}) \neq \emptyset, \forall \boldsymbol{d} \in \mathscr{D}\}$ where the regret $\mathrm{Reg}(\boldsymbol{x})$ of the first-stage commitment decision $\boldsymbol{x}$ is defined as

$$\mathrm{Reg}(\boldsymbol{x}) = \max_{d\in\mathscr{D}} \left\{ \min_{p\in\Omega(x,d)} \{c(\boldsymbol{x}) + b(\boldsymbol{p})\} - Q(\boldsymbol{d}) \right\}. \tag{6}$$

That is, the regret of $\boldsymbol{x}$ is the worst-case difference between the total production cost of choosing $\boldsymbol{x}$ before knowing $\boldsymbol{d}$ (i.e., $\min_{p\in\Omega(x,d)}\{c(\boldsymbol{x})+b(\boldsymbol{p})\}$) and the total production cost $Q(\boldsymbol{d})$ of a UC with the foresight of demand realization, where $Q(\boldsymbol{d})$ is defined as $Q(\boldsymbol{d})=\min_{x,p}\{c(\boldsymbol{x})+b(\boldsymbol{p}):\boldsymbol{x}\in\mathscr{F},\boldsymbol{p}\in\Omega(\boldsymbol{x},\boldsymbol{d})\}$.

The inner LO $\min_{p\in\Omega(x,d)}\{c(\boldsymbol{x})+b(\boldsymbol{p})\}$ can be dualized to a maximization problem just as we did in (4). The term $-Q(d)$ in (6) can also be easily rewritten as a maximization problem, given $Q(d)$ is defined by minimization problem. Therefore, $\mathrm{Reg}(\boldsymbol{x})$ in (6) can be reformulated as a standard max − min problem. The solution method developed in Section 2.3 can be applied to solve the resulting two-stage robust model.

3.3 From two-stage to multistage robust UC models

The two-stage robust UC model (2) ignores non-anticipativity in the dispatch, which dictates that the dispatch decision at hour t should only depend on realizations of uncertainty up to t. In a recent work [21], the authors show that respecting the non-anticipativity condition in the dispatch process is crucial for managing a system with restricted ramping capability, which has become a limiting factor in today's power systems with growing penetration of intermittent generation. The paper proposes a multistage robust UC model, which utilizes decision rules, in particular, affine policy, for the multistage dispatch process and develops efficient cutting-plane algorithms to solve the resulting large-scale robust optimization problem. For the first time, multistage robust UC problems can be solved for power networks of more than 2000 buses within a realistic time framework for applications in the day-ahead electricity markets. Furthermore, on realistic test cases, the simplified affine policy proposed in the paper performs surprisingly well — usually within 1% of the true optimal multistage policy.

4 Robust optimization in real-time operation and long-term planning

Robust optimization models are also proposed for the real-time dispatch and long-term planning problems. In the following, we outline some of the recent works in these areas.

A static robust optimization model is proposed for the look-ahead economic dispatch problem [33], where novel statistical models for wind forecast is incorporated into the robust ED model. In [29], a static robust ED model is proposed for managing system ramping capability, which is shown to outperform the recent development in deterministic look-ahead ED with ramping products [23].

In [20], a new two-stage robust optimization model with a new type of dynamic uncertainty sets is proposed for the multi-period dispatch problem. The dynamic uncertainty sets for wind power incorporate linear autoregression models of wind speeds at neighboring wind farms, so that the temporal and spatial correlations of wind speeds are captured. It is shown in [20] that the robust ED model with dynamic uncertainty sets can pareto dominates the performance of the robust ED model with the traditional budgeted uncertainty set (3) in both the average and variablity of the operational costs.

The fundamental two-stage robust optimization model (2) is applied to the real-time dispatch of automatic generation control (AGC) units in [37], where the first-stage decision is the dispatch of normal generation units to satisfy a nominal demand and then the second-stage problem dispatches automatic generation units to respond to demand fluctuations. An affine policy based robust optimization model is proposed for the AGC dispatching in [13], where the second-stage dispatch decision of the AGC units assumes to be an affine function of the uncertain load. as an approximation to the fully adaptive policy in (2), which makes the second-stage problem easier to solve. Affine decision rules are also proposed for managing reserves in the power system [31].

The two-stage robust optimization model (2) has also been applied to the long-term transmission network expansion planning problem in [14], where the uncertainties in renewable generation and loads are considered, and to the generation expansion planning problem in [10]. Adaptive robust models using affine decision rules are proposed for capacity expansion planning in [22].

5 Closing Remarks

This chapter gives a brief review of some of the recent developments in applying robust optimization methodology to power systems operation and planning under uncertainty. Many interesting directions, such as the integration of robust models into the electricity markets, multi-stage optimization for handling systems with high penetration of wind and solar power, and long-term investment are important questions, are open for further investigation.

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